



Statistics of radiation emitted from a quantum point contact

A. V. Lebedev,^{1,2} G. B. Lesovik,² and G. Blatter¹

¹*Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland*

²*L. D. Landau Institute for Theoretical Physics, RAS, 117940 Moscow, Russia*

(Received 14 December 2009; revised manuscript received 8 March 2010; published 8 April 2010)

We analyze the statistics of the electromagnetic radiation emitted from electrons pushed through a quantum point contact. We consider a setup implemented in a two-dimensional (2D) electron gas where the radiation manifests itself in terms of 2D-plasmons/photons emitted from electrons scattered at the point contact. The bosonic statistics of the photons compete with the fermionic statistics of the electrons; as a result, the quantum point contact emits nonclassical radiation with a statistics which can be tuned from bunching to anti-bunching by changing the driving voltage. Our perturbative calculation of the irreducible two-photon probability correlator provides us with information on the statistical nature of the emitted photons and on the underlying electronic current flow.

DOI: [10.1103/PhysRevB.81.155421](https://doi.org/10.1103/PhysRevB.81.155421)

PACS number(s): 73.50.Td, 42.50.Ar, 73.23.-b

I. INTRODUCTION

The interest in photon radiation from a quantum point contact (QPC) is twofold: on the one hand, the quantum point contact acts as a source of nonclassical light.^{1,2} While a classical current produces photons with Poisson statistics,³ the current pushed through a QPC can be tuned to radiate photons with either super or subpoissonian statistics,^{1,2} testifying for the bosonic nature of the photons (bunching) or the fermionic statistics (antibunching) of the underlying electrons radiating these photons, e.g., as “Bremsstrahlung” radiation in back-reflection processes. On the other hand, the radiation statistics contains information on the statistics of current fluctuations across the QPC—as such, the photodetector serves as a tool to probe the nearby current flow in the mesoscopic wire.^{4,5} Both themes have attracted considerable interest, be it in the context of the counting statistics of electrons in phase coherent mesoscopic conductors,^{6–8} where higher-order correlators carry the signatures of interactions, or be it related to the search for new sources of nonclassical radiation.^{1,2}

The classic photodetection theory for optical photons goes back to Glauber³ and is based on a threshold detector, where each (sufficiently energized) photon induces an electronic cascade generating a counting signal. The theory has been widely applied in the optical regime where threshold energies reside in the eV regime. The analysis of the photon counting statistics in Refs. 1 and 2 has been based on this photodetection theory and has concentrated on the regime, where the energy of emitted photons is larger than $eV/2$, with V the voltage bias applied to the quantum point contact. For this situation, the emitted photons are antibunched, since they are produced by different electrons scattered by a QPC.

The application of Glauber’s theory to mesoscopic setups is not straightforward, however, as typical photons generated in a mesoscopic structure have energies in the GHz regime and it may be difficult to construct a stable photodetector with a correspondingly small threshold energy. Furthermore, the emission rate of free photons scales with the small factor $\alpha(v_F/c)^2 \sim 10^{-8}$, where α is the fine structure constant, v_F is the Fermi velocity, and c the velocity of light, rendering the

observation of free photons practically impossible.

On the other hand, within a mesoscopic context, the low-energy exchange between a quantum point contact and the electromagnetic environment can be studied with a double-dot detector.^{9,10} An experimental setup has been introduced recently,¹¹ telling apart the different statistics of the low-energy (GHz) radiation from a thermal (50 Ω) resistor and a coherent (microwave) source. Furthermore, the recent experimental test of high-frequency shot noise in a quantum point contact¹² also provides a technique allowing for the generation and analysis of nonclassical photons in the microwave domain. Below, we study a setup inspired by these recent experiments,¹² where plasmons emitted from the QPC are transformed into electromagnetic modes and then analyzed in a power detector, cf. Figure 1. The emission rate for plasmons is enhanced over that of free photons by a factor $(c/v_{pl})^3 \sim 10^6$, where v_{pl} denotes the plasmon velocity; this gain in signal has to be preserved by proper impedance matching of the waveguides and the quantum point contact. Apart from determining the coupling between the QPC current and the bosonic electromagnetic modes, we can ignore

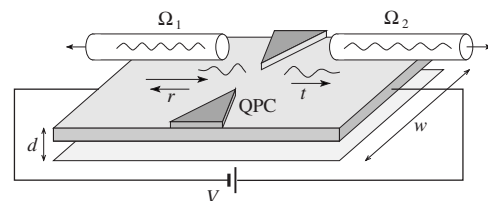


FIG. 1. Quantum point contact (QPC) fabricated through gates constricting the electron flow in a two-dimensional electron gas (2DEG). We consider a quasi-1D setup where the width w of the 2DEG allows propagation of one transverse mode; d is the distance to the back gate. The voltage (V) driven quantum point contact emits 2D plasmons with frequencies Ω_1 and Ω_2 due to electron backscattering (t and r denote transmission and reflection amplitudes). These plasmons are picked up by waveguides transmitting the signal (photons) to the measurement setup. The measurement of the one- $[P_1(\Omega)]$ and two-photon $[P_2(\Omega_1, \Omega_2)]$ emission probabilities allows to characterize the radiation and provides information on the fourth-order current correlator.

the difference between longitudinal density waves (plasmons) in the two-dimensional electron gas (2DEG) and transverse electromagnetic waves (photons) propagating in free space; in the following, we will refer to these bosonic modes as photons.

Below, we present a perturbative calculation (in the QPC–electromagnetic field coupling) of the one- $[P_1(\Omega)]$ and two-photon $[P_2(\Omega_1, \Omega_2)]$ probabilities for emission at given frequencies Ω , $\Omega_{1,2}$ and during an arbitrary but fixed time t_0 . Within our perturbative approach, we calculate the irreducible probability correlator $\bar{P}_2(\Omega_1, \Omega_2) = P_2(\Omega_1, \Omega_2) - P_1(\Omega_1)P_1(\Omega_2)$. The sign of this quantity then tells us about the character of the emitted radiation, bunching or antibunching. E.g., a positive sign indicates that photons prefer to be emitted simultaneously rather than independently and thus are bunched, while for a negative sign the opposite situation of anti-bunching prevails; for $\bar{P}_2(\Omega_1, \Omega_2) = 0$ the two photons are emitted independently. As we will show below, changing the bias on the quantum point contact will allow to change the statistics from antibunched fermionic type to bunched bosonic type. Furthermore, for different frequencies $\Omega_1 \neq \Omega_2$, the quantity $\bar{P}_2(\Omega_1, \Omega_2)$ is directly proportional to the irreducible fourth-order current correlator and hence its measurement provides valuable insight into the fluctuation statistics of the current across the quantum point contact.

In the following, we first define the model and find the expressions for the probability densities $p_1(\Omega)$ and $p_2(\Omega_1, \Omega_2)$ expressed through current correlators within a perturbative expansion, see Sec. II. In a second step, these probability densities are rewritten through the second and fourth order irreducible noise correlators $S^{(2)}$ and $S^{(4)}$. In Sec. III, we combine results for the probability densities and the noise correlators to find the single photon emission probability $P_1(\Omega) = p_1(\Omega)d\Omega$, the correlated two-photon emission probability $P_2(\Omega_1, \Omega_2) = p_2(\Omega_1, \Omega_2)d\Omega_1d\Omega_2$, as well as the irreducible quantity $\bar{P}_2(\Omega_1, \Omega_2)$; we identify the regimes of antibunching at intermediate voltages and the competition between anti-bunching and bunching at high voltage, when one electron has sufficient energy to emit two (bunched) photons. In Sec. IV, we discuss the interrelation between the fourth-order current correlator and the irreducible correlator $\bar{P}_2(\Omega_1, \Omega_2)$ and we conclude in Sec. V.

II. PHOTON EMISSION PROBABILITIES

In this section, we derive the formal expressions for the probability densities $p_1(\Omega)$ and $p_2(\Omega_1, \Omega_2)$ to emit one or two photons (plasmons) at frequencies Ω , $\Omega_{1,2}$ during a given time and express the results through the second and fourth-order current correlators. We begin with a short sketch describing the origin of the interaction Hamiltonian

$$\hat{H}_{\text{int}} = -\frac{1}{c} \int d^3r \hat{\mathbf{j}}(\mathbf{r}) \cdot \hat{\mathbf{A}}(\mathbf{r}), \quad (1)$$

where $\hat{\mathbf{j}}(\mathbf{r})$ describes the electronic current density through the QPC and $\hat{\mathbf{A}}(\mathbf{r})$ is the electromagnetic field associated with the plasmon modes.

The 2D plasmons are collective bosonic modes describing the coupled modulation of the electronic density and the electric field. The displacement field $\mathbf{u}(\mathbf{R}, t)$ of electrons generates the density modulation $\delta n(\mathbf{R}, t) = n(\mathbf{R}, t) - n_0 = n_0 \nabla \cdot \mathbf{u}(\mathbf{R}, t)$, where n_0 is the average electron density and $\mathbf{R} = \{x, y\}$. The modulation of the electron charge generates a three-dimensional electrostatic potential obeying Poisson's equation $\Delta \varphi(\mathbf{r}, t) = 4\pi e \delta n(\mathbf{r}, t) \delta(z)$. The electrostatic energy of the electron gas is given by $E_{\text{els}} = -e \int d^2\mathbf{R} \delta n(\mathbf{R}, t) \varphi(x, y, z=0, t)$, while the kinetic energy of the electrons associated with the density wave is given by $E_{\text{kin}} = (n_0 m / 2) \int d^2\mathbf{R} \dot{\mathbf{u}}^2(\mathbf{R}, t)$. The total Hamiltonian is a quadratic function of the displacement field \mathbf{u} and can be quantized in the usual manner. For a 2DEG in free space (e.g., electrons trapped on a liquid Helium surface¹³), this procedure results in a square root energy-momentum dispersion $\omega_k \propto \sqrt{k}$ of plasmon excitations. For the case where the electric field is screened,^{14,15} e.g., by a metallic gate separated from the 2DEG by a distance d , the plasmonic energy dispersion assumes a linear form $\omega_k = v_{\text{pl}} k$ for $kd \ll 1$, with v_{pl} the plasmon velocity.

The scattering of the electric current flowing through the QPC causes an additional density modulation $\delta n_j(\mathbf{R}, t)$ of the 2DEG. As a result, quantizing the plasmon modes one has to solve the Poisson equation due to the combined charge fluctuations, $\Delta \varphi(\mathbf{r}, t) = 4\pi e [\delta n_{\text{pl}}(\mathbf{R}, t) + \delta n_j(\mathbf{R}, t)] \delta(z)$, resulting in the electrostatic energy

$$E_{\text{els}} = -e \int d^2\mathbf{R} [\delta n_{\text{pl}}(\mathbf{R}, t) + \delta n_j(\mathbf{R}, t)] \times [\varphi_{\text{pl}}(\mathbf{R}, t) + \varphi_j(\mathbf{R}, t)],$$

where φ_{pl} and φ_j are the electrostatic potentials associated with the density modulations δn_{pl} and δn_j , respectively. The terms $\propto \varphi_{\text{pl}} \delta n_{\text{pl}}$ and $\propto \varphi_j \delta n_j$ describe the plasmon energy and the inductive energy of the external current, respectively, while the cross terms $\propto [\varphi_j \delta n_{\text{pl}} + \varphi_{\text{pl}} \delta n_j]$ quantify the interaction between the plasmons and the external current density.

Rather than solving the above coupled problem in detail, we note that the Fermi velocity v_F is typically much smaller than that of plasmons, $v_F / v_{\text{pl}} \ll 1$, and we have a separation of scales between the two density waves; as a consequence, the resulting interaction between the plasmon modes and the electric current is small and well localized near the QPC. Furthermore, we can (using a gauge transformation $\varphi \rightarrow \mathbf{A}$) rewrite the interaction into the form Eq. (1) and insert the separate fields $\mathbf{j}(\mathbf{r})$ and $\hat{\mathbf{A}}(\mathbf{r})$ describing the electrons and the plasmons.

To fix ideas, we consider plasmon modes propagating along the one-dimensional wire (the x direction) and restrict ourselves to the lowest transverse mode only (we assume translation invariance along the y direction). The vector potential $\hat{\mathbf{A}}(x)$ then has only a component along x ,

$$\hat{A}_x(x) = \sum_k ik \gamma \left(\frac{2\pi \hbar c^2}{\omega_k L_{\text{quant}}} \right)^{1/2} (\hat{c}_k e^{ikx} - \hat{c}_k^\dagger e^{-ikx}), \quad (2)$$

where \hat{c}_k^\dagger (\hat{c}_k) are bosonic creation (annihilation) operators for the (longitudinal) plasmon modes with wave vector k . The presence of a nearby backgate¹⁵ (with distance d) gen-

erates a linear dispersion $\omega_k = v_{\text{pl}}k$ (Ref. 15) and $\gamma \sim \sqrt{d/w}$ is a geometrical factor with w the width of the 2DEG. In the following, we set the quantization length L_{quant} equal to unity.

With the current operator $\hat{j}_x(\mathbf{r}) = \hat{I}(x)\delta^3(\mathbf{r} - x\mathbf{e}_x)$ in the wire, the Hamiltonian \hat{H}_{int} in the interaction representation takes the form

$$\hat{H}_{\text{int}}(t) = -i\gamma \sum_k k \left(\frac{2\pi\hbar}{\omega_k} \right)^{1/2} (\hat{c}_k \hat{I}_k(t) e^{-i\omega_k t} - \hat{c}_k^\dagger \hat{I}_k^\dagger(t) e^{i\omega_k t}), \quad (3)$$

where $\hat{I}_k(t)$ is the spatial average of the current $\hat{I}(x, t)$ over the coupling region of the plasmon described by the kernel $f(x)$ with extension L ,

$$\hat{I}_k(t) = \int dx \hat{I}(x, t) f(x) e^{ikx}. \quad (4)$$

In a typical situation, the frequency of the excited plasmons lies in the GHz range, with the velocity v_{pl} roughly 100 times slower than the speed of light c . The corresponding wavelength $\lambda_{\text{pl}} \sim 100 \mu\text{m}$ then is much larger than the size $\sim 0.1 \mu\text{m}$ of the quantum point contact. Assuming a coupling region $0.1 \mu\text{m} < L < \lambda_{\text{pl}}$, we can ignore the k dependence in $\hat{I}_k(t)$.

In a specific experiment, the dimensionless coupling $\alpha \propto (e^2/\hbar v_{\text{pl}})(v_{\text{F}}/v_{\text{pl}})^2$ deriving from the above analysis, cf. Eq. (22), is reduced when plasmons are converted to photons propagating in the waveguides for further analysis. In the experiment of Ref. 12, the wavelength of the plasmons involved is even larger than the sample's size; in this case the charge fluctuations generated at the QPC are directly coupled into the transmission lines. Hence, the precise numerical value of the coupling between the QPC and the photons analyzed in the measurement setup has to be determined for the specific situation at hand. Furthermore, we note that the results derived below hold true, up to a numerical factor of order unity, for the direct emission of (long wavelength) photons as well; a corresponding setup with LC circuit pickups and quadratic detectors is sketched in Fig. 2. For the direct photonic emission, one needs to replace the plasmon wave velocity v_{pl} by the speed of light c and set the geometry factor $\gamma = 1$. The dimensionless coupling α then is reduced by the small factor $(v_{\text{pl}}/c)^3$ (with $v_{\text{pl}}/c \sim 10^{-2}$ typically) when three-dimensional photons are emitted into space. Similarly, the plasmon velocity v_{pl} appearing in our expressions below should be replaced by the velocity of light in the transmitting medium. In the following, we will refer to the electromagnetic modes as “photons.”

We assume that initially, at time $t = -t_0 < 0$, no photons are excited and the state of the total system (QPC and bosonic modes) is described by the factorized density matrix $\hat{\rho}(-t_0) = \hat{\rho}_{\text{pl}}(-t_0) \otimes \hat{\rho}_{\text{QPC}}(-t_0)$. At a later time $t = 0$, the probability density $p_n(\Omega_1, \dots, \Omega_n)$ to find n photons with frequencies $\Omega_1, \dots, \Omega_n$ can be defined in terms of the time-ordered evolution operator $\hat{S}(0, -t_0) = T \exp[-(i/\hbar) \int_{-t_0}^0 \hat{H}_{\text{int}}(t') dt']$,

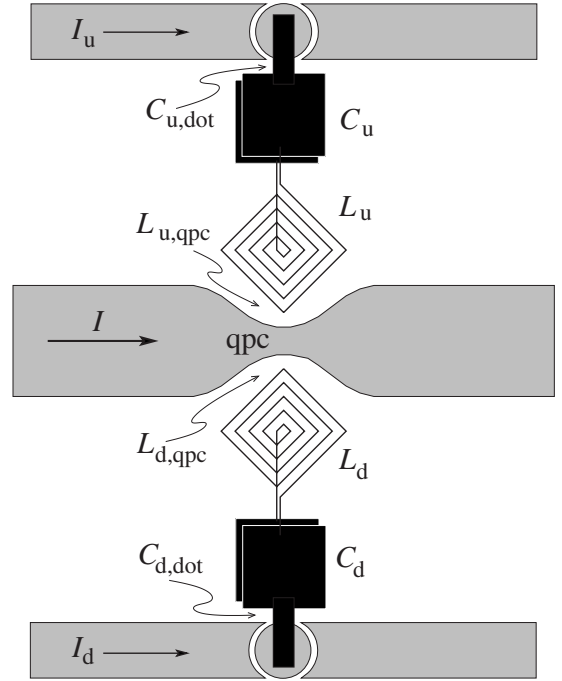


FIG. 2. Setup to probe the emission of photons from a QPC into free space. The photons are picked up by the LC circuits “up” and “down,” which are inductively ($L_{u(d),\text{qpc}}$) coupled to the QPC and capacitively ($C_{u(d),\text{dot}}$) coupled to the quantum dots. The latter are tuned to minimal or maximal transmission and thus operate as quadratic detectors. Photons absorbed in the LC-circuits modulate the charge on the capacitors $C_{u(d)}$ and, through coupling to the quantum dots, the probe currents I_u and I_d .

$$p_n = \text{Tr}\{\hat{\mathcal{P}}_n(\Omega_1, \dots, \Omega_n) \hat{S}(0, -t_0) \hat{\rho}(0) \hat{S}^\dagger(0, -t_0)\}, \quad (5)$$

where $\hat{\rho}(t)$ is the time dependent density matrix of the electronic system (including scattering at the QPC, interactions between electrons, etc.), $\hat{\mathcal{P}}_n(\Omega_1, \dots, \Omega_n)$ is the projection operator on the state with n bosons of frequencies Ω_i , $i = 1, \dots, n$, and the trace is taken with respect to the total system. The Taylor expansion of the evolution operator $\hat{S}(0, t)$ up to the lowest nontrivial order provides us with the (one photon) probability density,

$$p_1(\Omega) = \gamma^2 \frac{\Omega}{\hbar v_{\text{pl}}^3} \int_{-t_0}^0 ds d\tau e^{i\Omega(\tau-s)} \langle \hat{I}(s) \hat{I}(\tau) \rangle, \quad (6)$$

where $\langle \dots \rangle$ denotes the average over the electronic system. The next term in the series generates the two-photon probability density

$$\begin{aligned} p_2(\Omega_1, \Omega_2) &= \frac{\gamma^4}{4} \frac{\Omega_1}{\hbar v_{\text{pl}}^3} \frac{\Omega_2}{\hbar v_{\text{pl}}^3} \int_{-t_0}^0 ds_1 ds_2 d\tau_1 d\tau_2 \\ &\times \langle T_- \{ \hat{I}(s_1) \hat{I}(s_2) \} T_+ \{ \hat{I}(\tau_1) \hat{I}(\tau_2) \} \rangle \\ &\times (e^{i\Omega_1(\tau_1-s_1)} e^{i\Omega_2(\tau_2-s_2)} + e^{i\Omega_1(\tau_1-s_2)} e^{i\Omega_2(\tau_2-s_1)} \\ &+ \Omega_1 \leftrightarrow \Omega_2), \end{aligned} \quad (7)$$

where T_+ and T_- are the time ordering operators in the forward and backward directions, respectively. In the following,

we assume $t_0 \gg \Omega^{-1}$, $\Omega_{1,2}^{-1}$ and regularize the time integrals in Eqs. (6) and (7) at the lower limit by introducing a small damping factor $\exp(-\eta|\tau|)$ with $\eta \ll \Omega$. Physically, η^{-1} corresponds to the decay time of the photon excitations propagating in the wave guide.

The above time integrals over current correlators can be expressed through the spectral power of current fluctuations. Assuming a stationary situation, the current correlators in Eqs. (6) and (7) depend only on relative times; the second-order noise correlator $S^{(2)}$ then can be defined through

$$S^{(2)}(\omega) = \int d\tau e^{-i\omega\tau} \langle \hat{I}(\tau) \hat{I}(0) \rangle, \quad (8)$$

and the fourth-order correlator $S^{(4)}$ reads

$$S^{(4)}(\omega_1, \omega_2, \omega_3) = \int d\tau_1 d\tau_2 d\tau_3 e^{-i\omega_1\tau_1 - i\omega_2\tau_2 - i\omega_3\tau_3} \times \langle \hat{I}(\tau_1 + \tau_2 + \tau_3) \hat{I}(\tau_2 + \tau_3) \hat{I}(\tau_3) \hat{I}(0) \rangle. \quad (9)$$

The noise correlators $S^{(2)}$ and $S^{(4)}$ involve *irreducible* current correlators, while the expressions for $p_1(\Omega)$ and $p_2(\Omega_1, \Omega_2)$ are expressed in terms of *reducible* quantities. Expressing the reducible correlator through irreducible ones,

$$\langle \hat{I}(s) \hat{I}(\tau) \rangle = \langle \hat{I}(s) \hat{I}(\tau) \rangle + \langle \hat{I}(s) \rangle \langle \hat{I}(\tau) \rangle, \quad (10)$$

and using the Fourier transform $\langle \hat{I}(s) \hat{I}(\tau) \rangle = \int (d\omega/2\pi) \exp[i\omega(s-\tau)] S^{(2)}(\omega)$, we can perform the time integrals in the expression Eq. (6) for $p_1(\Omega)$. In the stationary regime, the average current $\langle I(t) \rangle = \bar{I}$ through the QPC is independent of time and we obtain the intermediate form

$$p_1(\Omega) = \gamma^2 \frac{\Omega}{\hbar v_{\text{pl}}^3} \left[\int \frac{d\omega}{2\pi} \frac{S^{(2)}(\omega)}{(\omega - \Omega)^2 + \eta^2} + \frac{\bar{I}^2}{\Omega^2 + \eta^2} \right]. \quad (11)$$

In the limit $\eta \ll \Omega$, we can approximate the Lorentzian by a δ function, $\eta/[(\omega - \Omega)^2 + \eta^2] \approx \pi \delta(\omega - \Omega)$, and carry out the integral over ω to arrive at the final result,

$$p_1(\Omega) = \gamma^2 \frac{\Omega}{\hbar v_{\text{pl}}^3} \left[\frac{S^{(2)}(\Omega)}{2\eta} + \frac{\bar{I}^2}{\Omega^2 + \eta^2} \right]. \quad (12)$$

Furthermore, the contribution from the irreducible part of the current correlator to $p_1(\Omega)$ is a factor $\Omega/\eta \gg 1$ larger than the contribution from the reducible part $\propto \bar{I}^2$, as follows from the estimate $S^{(2)}(\Omega) \sim \bar{I}^2/\Omega$; we will drop the reducible part in our further analysis below.

Expressing $p_2(\Omega_1, \Omega_2)$ through irreducible correlators is more involved: below, we keep only those terms of the reducible fourth-order current correlator which give nonvanishing contributions to p_2 in the limit $\eta \ll \Omega$,

$$\begin{aligned} \langle \hat{I}(s_2) \hat{I}(s_1) \hat{I}(\tau_1) \hat{I}(\tau_2) \rangle &= \langle \hat{I}(s_2) \hat{I}(s_1) \hat{I}(\tau_1) \hat{I}(\tau_2) \rangle \\ &+ \langle \hat{I}(s_2) \hat{I}(\tau_2) \rangle \langle \hat{I}(s_1) \hat{I}(\tau_1) \rangle + \dots, \end{aligned} \quad (13)$$

where we assume $\tau_1 > \tau_2$, $s_1 > s_2$ and ... denotes terms with

different time orderings as well as third-order cumulants providing irrelevant contributions. The probability distribution p_2 can then be expressed through the noise correlators $S^{(4)}$ and $S^{(2)}$; the contribution from the reducible part of the fourth-order current correlator reads

$$p_2^{(2)}(\Omega_1, \Omega_2) = \gamma^4 \frac{\Omega_1}{\hbar v_{\text{pl}}^3} \frac{\Omega_2}{\hbar v_{\text{pl}}^3} \frac{S^{(2)}(\Omega_1) S^{(2)}(\Omega_2)}{4\eta^2} \times \left[1 + \frac{\eta^2}{(\Omega_1 - \Omega_2)^2 + \eta^2} \right]. \quad (14)$$

This term describes the excitation of two photons due to the uncorrelated scattering of independent electrons. For two bosons with equal frequency $|\Omega_1 - \Omega_2| \ll \eta$, this contribution is enhanced by a factor 2 as compared with the probability to emit two bosons with different frequencies [see Eq. (12)],

$$p_2^{(2)}(\Omega_1, \Omega_2) = \begin{cases} p_1(\Omega_1) p_1(\Omega_2), & |\Omega_1 - \Omega_2| \gg \eta, \\ 2p_1^2(\Omega_1), & |\Omega_1 - \Omega_2| \ll \eta. \end{cases} \quad (15)$$

This enhancement is a quantum mechanical time-interference effect: for $|\Omega_1 - \Omega_2| \ll \eta$ we cannot know which boson was emitted first during the measurement time η^{-1} and the amplitudes of both alternatives have to be added, resulting in a constructive interference between them.

Next, we concentrate on the contribution $p_2^{(4)}$ arising from the irreducible part of the fourth-order current correlator in Eq. (13). After integration over times, we arrive at the expression

$$\begin{aligned} p_2^{(4)}(\Omega_1, \Omega_2) &= \gamma^4 \frac{\Omega_1}{\hbar v_{\text{pl}}^3} \frac{\Omega_2}{\hbar v_{\text{pl}}^3} \\ &\times \int \frac{d\omega_1 d\omega_2 d\omega_3}{(2\pi)^3} \frac{S^{(4)}(\omega_1, \omega_2, \omega_3)}{(\omega_2 - \Omega_1 - \Omega_2)^2 + 4\eta^2} \\ &\times \left[\frac{1}{(\omega_1 - \Omega_1 - i\eta)(\omega_3 - \Omega_1 + i\eta)} \right. \\ &\left. + \frac{1}{(\omega_1 - \Omega_1 - i\eta)(\omega_3 - \Omega_2 + i\eta)} + \Omega_1 \leftrightarrow \Omega_2 \right]. \end{aligned} \quad (16)$$

Combining Eqs. (14) and (16) and approximating the Lorentzian in Eq. (16) by a δ function, one finally arrives at the expression for the probability density to emit two photons,

$$p_2(\Omega_1, \Omega_2) = \gamma^4 \frac{\Omega_1}{\hbar v_{\text{pl}}^3} \frac{\Omega_2}{\hbar v_{\text{pl}}^3} \frac{1}{4\eta} Q^{(4)}(\Omega_1, \Omega_2) + p_2^{(2)}(\Omega_1, \Omega_2), \quad (17)$$

with

$$\begin{aligned} Q^{(4)}(\Omega_1, \Omega_2) &= \int \frac{d\omega_1 d\omega_3}{(2\pi)^2} \left[\frac{S^{(4)}(\omega_1, \Omega_1 + \Omega_2, \omega_3)}{(\omega_1 - \Omega_1 - i\eta)(\omega_3 - \Omega_1 + i\eta)} \right. \\ &\left. + \frac{S^{(4)}(\omega_1, \Omega_1 + \Omega_2, \omega_3)}{(\omega_1 - \Omega_1 - i\eta)(\omega_3 - \Omega_2 + i\eta)} + \Omega_1 \leftrightarrow \Omega_2 \right]. \end{aligned} \quad (18)$$

The resulting probability densities Eqs. (12) and (17) involve second-order current correlators at positive frequencies and integrals over fourth-order current correlators with integration kernels concentrated near positive frequencies as well. This feature is easily understood: initially there are no bosonic excitations, hence within our lowest-order calculation the only processes contributing to $p_1(\Omega)$ and $p_2(\Omega_1, \Omega_2)$ are due to photon emission—reabsorption processes involving negative frequency correlators show up only within a higher-order analysis.

III. PHOTON COUNTING STATISTICS FOR A QUANTUM CURRENT

It is a well known result due to Glauber³ that a *classical* current $I(t)$ produces a coherent state of the electromagnetic field, with the width of the Poisson statistics of photo counts for each mode given by the corresponding Fourier coefficient $I(\omega)$ of the current. This implies that photons are emitted independently, i.e., the joint probability to emit two photons with frequencies Ω_1 and Ω_2 is in fact a product of single photon probabilities, $P_2(\Omega_1, \Omega_2) = P_1(\Omega_1)P_1(\Omega_2)$.

For a *quantum* current the fluctuations appear due to the scattering of separate electrons. In the scattering process, the electrons experience an acceleration and emit *Bremsstrahlung* radiation. The fermionic statistics induces a correlated flow of the electrons incoming from the voltage biased reservoir and separate electrons are scattered one by one. The photons emitted in the scattering of separate electrons then inherit their fermionic correlations. On the other hand, one electron may produce several photons during the scattering event, in which case these photons are bunched. The deviation from poissonian statistics thus is a competition between these two processes.

Using the results of the previous section, we can find the probability $P_1(\Omega) = p_1(\Omega)d\Omega$ for the emission of photons in the frequency interval $d\Omega$ around Ω and $P_2(\Omega_1, \Omega_2) = p_2(\Omega_1, \Omega_2)d\Omega_1d\Omega_2$ for the emission of two photons at given frequencies within the nonoverlapping frequency windows $d\Omega_1$ and $d\Omega_2$ around Ω_1, Ω_2 ; for overlapping frequency intervals $\Omega_1 \approx \Omega_2 = \Omega$, the determination of $P_2(\Omega)$ requires proper integration of the density $p_2(\Omega_1, \Omega_2)$.

While the results on the one and two-photon processes provide us with only limited information on the photo counting statistics, they are nevertheless sufficient to tell us about important quantum signatures in the radiation. In particular, the statistical properties of the emitted photons is conveniently described by the irreducible probability correlator,

$$\bar{P}_2(\Omega_1, \Omega_2) = P_2(\Omega_1, \Omega_2) - P_1(\Omega_1)P_1(\Omega_2). \quad (19)$$

A positive sign of $\bar{P}_2(\Omega_1, \Omega_2)$ indicates that photons are *bunched*, i.e., they are preferentially emitted simultaneously. In the opposite case of negative correlations, $\bar{P}_2(\Omega_1, \Omega_2) < 0$, the photons are *antibunched*, implying that the probability to find the second photon emitted right after the first is suppressed.

A. Single photon emission

In the following, we assume that the emitted radiation is collected from a finite region of length $L < \lambda_{pl}$ and we average the current operators in Eq. (4) using a specific kernel of the form $f(x) = \exp(-|x|/L)$. First, we concentrate on the single photon probability $P_1(\Omega)$. Inserting the ($T=0$) second-order noise correlator¹⁶

$$S_{x_1x_2}^{(2)}(\Omega) = \frac{e^2}{\pi} TR e^{i(\Omega/v_F)[|x_2| - |x_1|]} (\omega_0 - \Omega) \Theta(\omega_0 - \Omega), \quad (20)$$

into Eq. (12) [$\Theta(x)$ is the Heaviside function], one arrives at the result for the single photon emission probability in the form,

$$P_1(\Omega) = \frac{2\alpha}{\pi} TR \left(\frac{\omega_0 - \Omega}{\Omega} \right) \frac{\Theta(\omega_0 - \Omega)}{1 + (\Lambda/L)^2} \frac{1}{\eta} d\Omega, \quad (21)$$

with $\omega_0 = eV/\hbar$ the voltage frequency, $\Lambda = v_F/\Omega \sim 100 \mu\text{m}$ the characteristic spatial scale of the current fluctuations, and α is the dimensionless parameter,

$$\alpha = \gamma^2 \left(\frac{e^2}{\hbar v_{pl}} \right) \left(\frac{v_F}{v_{pl}} \right)^2 \sim \frac{\gamma^2}{100}. \quad (22)$$

Note that with d and w of order $1 \mu\text{m}$ (Ref. 15) the geometry factor γ is of order unity. On the other hand, the parameter α is reduced due to the residual impedance mismatch between the quantum point contact and the waveguides, cf. Figure 1.

The probability $P_1(\Omega)$ describes the Bremsstrahlung photo emission due to electron scattering at the barrier. Although one may expect the probability $P_1(\Omega)$ to increase for a more effective scatterer with $T \rightarrow 0$, the result is in fact proportional to $T(1-T)$ and vanishes in the tunneling limit. This peculiarity of the Bremsstrahlung appears due to the Fermi statistics: the electron coming in from the biased reservoir has to relax to a state with a lower energy after the photon emission. Since all electron scattering states of the biased reservoir with lower energy are filled, the only possibility for the electron to decay is into an empty scattering state of the unbiased reservoir. This process requires tunneling of the electron through the barrier and hence the probability $P_1(\Omega)$ turns out proportional to T . The sharp suppression of $P_1(\Omega)$ when the photon energy $\hbar\Omega$ is larger than the applied bias eV can be explained in the same way: an electron cannot find an empty state to emit such a “high-energy” photon.

B. Correlated photon emission

Next, we analyze the two-photon probability $P_2(\Omega_1, \Omega_2)$, see Eq. (17), which involves irreducible fourth-order noise correlators. Below, we consider the case of an *extended* interaction region, where the signal is collected over a region of size L larger than the characteristic spacial scale $\Lambda = v_F/\Omega_{1,2} \sim 100 \mu\text{m}$ of current fluctuations, where we consider typical Fermi velocities $v_F \sim 10^7 \text{ cm/s}$ in GaAs heterostructures and frequencies in the GHz regime. The calcula-

tion of the expression $Q^{(4)}(\Omega_1, \Omega_2)$ in Eq. (18) is carried out in the Appendix, cf. Equation (A19) for the final result; the probability $P_2(\Omega_1, \Omega_2)$ for two-photon emission into non-overlapping frequency intervals then reads

$$P_2(\Omega_1, \Omega_2) = \frac{4\alpha^2}{\pi\eta} [RT(T-R)^2\Theta(\omega_0 - \Omega_\Sigma)(\omega_0 - \Omega_\Sigma) - 2(RT)^2\Theta(\omega_0 - \Omega_{>})(\omega_0 - \Omega_{>})] \frac{d\Omega_1 d\Omega_2}{\Omega_1 \Omega_2} + \frac{4\alpha^2}{(\pi\eta)^2} (RT)^2\Theta(\omega_0 - \Omega_1)\Theta(\omega_0 - \Omega_2)(\omega_0 - \Omega_1) \times (\omega_0 - \Omega_2) \left[1 + \frac{\eta^2}{(\Omega_1 - \Omega_2)^2 + \eta^2} \right] \frac{d\Omega_1 d\Omega_2}{\Omega_1 \Omega_2}, \quad (23)$$

where $\Omega_\Sigma = \Omega_1 + \Omega_2$ and $\Omega_{>} = \max\{\Omega_1, \Omega_2\}$. The first term $\propto 1/\eta$ in the above expression originates from the irreducible fourth-order noise correlator $Q^{(4)}$, while the second term $\propto 1/\eta^2$ is a contribution from the reducible part $p_2^{(2)}$ of the fourth-order current correlator, see Eq. (17).

For nonoverlapping frequency bands with $|\Omega_1 - \Omega_2| \gg \eta$, the second term in the bracket [...] in the reducible part is small and what remains is equal to the product $P_1(\Omega_1)P_1(\Omega_2)$ of single photon probabilities. Hence, the irreducible probability correlator \bar{P}_2 , Eq. (19), involves only the first irreducible term in Eq. (23),

$$\bar{P}_2(\Omega_1, \Omega_2) = \frac{4\alpha^2}{\pi\eta} [RT(T-R)^2\Theta(\omega_0 - \Omega_\Sigma)(\omega_0 - \Omega_\Sigma) - 2(RT)^2\Theta(\omega_0 - \Omega_{>})(\omega_0 - \Omega_{>})] \frac{d\Omega_1 d\Omega_2}{\Omega_1 \Omega_2}. \quad (24)$$

In the opposite situation where the frequency bands overlap, the density $p_2(\Omega_1, \Omega_2)$ has to be properly integrated over frequencies; the second term (of Lorentzian shape) in $p_2^{(2)}$, cf. Equation (14), then provides a contribution π/η . For a narrow frequency band $\delta\Omega/\Omega \ll 1$ far from the voltage frequency, $|\omega_0 - \Omega| \sim \Omega$, this reducible contribution dominates over the irreducible one originating from $Q^{(4)}$ and the irreducible probability correlator Eq. (19) is dominated by the reducible part of the fourth-order current correlator,

$$\bar{P}_2(\Omega) \approx \frac{4\alpha^2}{\pi\eta} (TR)^2\Theta(\omega_0 - \Omega)(\omega_0 - \Omega) \frac{\omega_0 - \Omega}{\Omega} \frac{\delta\Omega}{\Omega}. \quad (25)$$

The situation changes if one detects photons in the overlapping frequency band reaching the voltage frequency, $\Omega_1, \Omega_2 \in [\omega_0 - \Delta\Omega, \omega_0]$, $\Delta\Omega < \omega_0/2$. In this situation one needs to take the remaining integrals over the frequencies Ω_1, Ω_2 in Eq. (23) exactly. The result takes the form

$$P_2(\Omega) = -\frac{8\alpha^2}{3\pi} (RT)^2 \frac{(\Delta\Omega)^3}{\eta\Omega^2} + \frac{4\alpha^2}{3\pi} (RT)^2 \frac{(\Delta\Omega)^3}{\eta\Omega^2}, \quad (26)$$

where the first term originates from the irreducible part of the fourth-order current correlator, while the second term has its origin in the reducible contribution. We then find that the

irreducible contribution dominates in this regime.

In summary, the irreducible correlator \bar{P}_2 at different frequencies $\Omega_1 \neq \Omega_2$ involves a competition between bunching [first term in Eq. (24)] and antibunching [second term in Eq. (24)]. When the frequency intervals strongly overlap we encounter two regimes: (1) detecting the photon in a narrow frequency band far from the voltage frequency, $|\omega_0 - \Omega| \sim \Omega \gg \delta\Omega$, the reducible contribution of the current correlator always dominates thus resulting in bunched photon radiation; (2) measuring the photon in the frequency band $[\omega_0 - \Delta\Omega, \omega_0]$, $\Delta\Omega < \omega_0/2$, near the voltage frequency, the irreducible contribution of the current correlator for a single-channel conductor is two times larger than the reducible one in absolute values, resulting in anti-bunched radiation.

It is the irreducible correlator \bar{P}_2 at different frequencies $\Omega_1 \neq \Omega_2$ which is the most interesting quantity measuring the nontrivial correlator $Q^{(4)}$. Thus, in order to study the fourth-order electron noise effects on the photon statistics, one needs to be able to distinguish between the frequencies of the emitted photon during the measurement time η^{-1} .

We thus concentrate on the detection of photons with distinguishable frequencies. The probability correlator $\bar{P}_2(\Omega_1, \Omega_2)$, see Eq. (24), involves two terms with opposite signs, a positive one proportional to $\Theta(\omega_0 - \Omega_1 - \Omega_2)$ and a negative contribution $\propto \Theta(\omega_0 - \max\{\Omega_1, \Omega_2\})$. Applying a small voltage bias $\omega_0 < \max\{\Omega_1, \Omega_2\}$, the scattered electrons do not have the possibility to emit two photons with frequencies Ω_1 and Ω_2 due to the restriction of the Fermi statistics on the final electron state and hence $\bar{P}_2(\Omega_1, \Omega_2) = 0$.

In the intermediate voltage regime $\max\{\Omega_1, \Omega_2\} < \omega_0 < \Omega_1 + \Omega_2$, a single electron can emit only one photon, either of frequency Ω_1 or Ω_2 . Thus, the corresponding contribution $\propto \Theta(\omega_0 - \max\{\Omega_1, \Omega_2\})$ is due to the scattering of different (fermionically correlated) electrons. At zero temperature, these electrons arrive at the scatterer in perfect order with a time separation $\tau_V \sim h/eV$. Hence, the second photon is emitted only after the time τ_V , resulting in an antibunched radiation statistics and a negative probability correlator $\bar{P}_2(\Omega_1, \Omega_2) < 0$. When a single electron creates only one photon, the complexity of the emission process is reduced considerably, allowing for the determination of the full counting statistics of the emitted radiation.^{1,2}

Increasing the voltage beyond $\hbar(\Omega_1 + \Omega_2)$, the above single photon production is augmented by processes where one electron emits two photons in a single scattering event; this process generates bunched radiation and hence the corresponding term $\propto \Theta(\omega_0 - \Omega_1 - \Omega_2)$ is positive, see Eq. (24). The overall sign of the probability correlator $\bar{P}_2(\Omega_1, \Omega_2)$ then appears in a competition between the two processes creating photons individually or in pairs. Changing the parameters of the quantum point contact, one can control the relative weight of the two contributions. For example, at $T = 1/2$ the two-photon process $\propto TR(T-R)^2$ vanishes while the single-photon term $\propto 2(TR)^2$ is maximal, resulting in antibunched radiation in the whole two-photon voltage regime $\Omega_1 + \Omega_2 < \omega_0 < \Omega_1 + \Omega_2 + \min\{\Omega_1, \Omega_2\}$ (we consider three-photon processes involving at least one photon with frequency Ω_1 and one with Ω_2). Alternatively, in the tunneling limit $T \ll 1$, the

emission of bunched photons is the dominant process at $\omega_0 > \Omega_1 + \Omega_2$. A further increase of the bias voltage may lead to multiphoton production processes, where more than two photons are created by a single scattered electron. However, such processes appear only in higher orders of perturbation theory, while we have restricted ourselves to the fourth order in the coupling constant.

Finally, we analyze the case of a multichannel conductor in the diffusive regime. The probability correlator $\bar{P}_2(\Omega_1, \Omega_2)$ for nonoverlapping frequencies in the multichannel case can be obtained by summing the terms in Eq. (24) over the channel index n with appropriate values for the scattering coefficients T_n and R_n . The distribution of the transmission eigenvalues T in the diffusive conductor is given by the Dorokhov distribution function,¹⁷

$$\rho(T) = \frac{G}{2G_0 T} \frac{1}{\sqrt{1-T}}, \quad (27)$$

where G is the average conductance and $G_0 = 2e^2/h$. Averaging Eq. (24) over T , the probability correlator $\bar{P}_2(\Omega_1, \Omega_2)$ for the radiation emitted from the diffusive conductor is given by the expression (we assume $\Omega_1 \neq \Omega_2$),

$$\begin{aligned} \bar{P}_2(\Omega_1, \Omega_2) &= \frac{4\alpha^2 G}{\pi\eta G_0} \left[\frac{11}{105} \Theta(\omega_0 - \Omega_2)(\omega_0 - \Omega_2) \right. \\ &\quad \left. - \frac{4}{35} \Theta(\omega_0 - \Omega_>)(\omega_0 - \Omega_>) \right] \frac{d\Omega_1}{\Omega_1} \frac{d\Omega_2}{\Omega_2}. \end{aligned} \quad (28)$$

Quite remarkably, the result is negative, indicating antibunched photons, even in the two-photon regime $\omega_0 > \Omega_1 + \Omega_2$. This provides, besides the measurements of the average current and noise, another consistency test for the Dorokhov distribution function.

C. Measurement setup

Next, we relate the probabilities $P_1(\Omega)$ and $P_2(\Omega_1, \Omega_2)$ to physically measurable quantities. In a realistic experiment, see Ref. 12, a two-terminal quantum point contact is realized in a two-dimensional electron gas inserted between two transmission lines, cf. Figure 1. The excited plasmon excitations in the 2DEG induce an *ac*-electric current signal in the two (left and right) transmission lines. The transmitted signals are independently amplified by two cryogenic amplifiers and then passed through frequency filters (selecting proper frequencies $\Omega_{1,2}$) followed by quadratic detectors. Thus the signal taken at the end of each transmission line is proportional to the power emitted from the QPC,

$$\hat{W}_i = A_i \hbar \Omega_i \hat{n}(\Omega_i) d\Omega_i + w_i, \quad i = 1, 2, \quad (29)$$

where A_i is the amplification factor, $\hat{n}(\Omega) d\Omega$ is the photon production rate, i.e., the number of excited photons per unit time within the frequency band $d\Omega$, and w_i is a parasitic power due to the intrinsic noise of the amplifiers and the vacuum fluctuations of the bosonic mode. Given the life time (or detection time) $1/\eta$ of the photons, cf. Eqs. (12) and (17),

the photon production rate $\hat{n}(\Omega) d\Omega$ relates to the photon occupation number $\hat{N}(\Omega)$ via $\hat{n}(\Omega) d\Omega = \eta \hat{N}(\Omega)$.

Since the noise signals $w_{1,2}$ are not correlated for the different transmission lines, $\langle \delta w_1 \delta w_2 \rangle = 0$, $\delta w_i = w_i - \langle w_i \rangle$, the irreducible cross-correlator between the two transmission lines, $Q_{12} = \langle \hat{W}_1 \hat{W}_2 \rangle - \langle \hat{W}_1 \rangle \langle \hat{W}_2 \rangle$, involves only the irreducible cross-correlator of the photon occupation numbers emitted into each line,

$$Q_{12} = A_1 A_2 \hbar \Omega_1 \hbar \Omega_2 \eta^2 \langle \langle \hat{N}(\Omega_1) \hat{N}(\Omega_2) \rangle \rangle. \quad (30)$$

Next, we express the photon number correlator $\langle \langle \hat{N}(\Omega_1) \hat{N}(\Omega_2) \rangle \rangle$ through the probabilities $P_1(\Omega)$ and $P_2(\Omega_1, \Omega_2)$ calculated above. Up to fourth order in perturbation theory, we have

$$\langle \hat{N}(\Omega) \rangle \approx p_1(\Omega) d\Omega + \left[\int d\Omega' p_2(\Omega, \Omega') \right] d\Omega, \quad (31)$$

$$\langle \hat{N}(\Omega_1) \hat{N}(\Omega_2) \rangle \approx p_2(\Omega_1, \Omega_2) d\Omega_1 d\Omega_2, \quad (32)$$

where the second term in $\langle \hat{N}(\Omega) \rangle$ is a higher order correction. As a result, the irreducible cross correlator for the photon number then assumes the simple form,

$$\langle \langle \hat{N}(\Omega_1) \hat{N}(\Omega_2) \rangle \rangle = P_2(\Omega_1, \Omega_2) - P_1(\Omega_1) P_1(\Omega_2), \quad (33)$$

involving only the probabilities $P_2(\Omega_1, \Omega_2)$ and $P_1(\Omega_1)$, $P_1(\Omega_2)$; other terms are of higher order in the coupling constant. The cross correlator of the emitted power between two transmission lines then is proportional to the probability correlator $\bar{P}_2(\Omega_1, \Omega_2)$, see Eq. (19), and the sign of the power cross correlator directly characterizes the statistics of the emitted radiation.

Finally, we connect our results with those of Beenakker and Schomerus,^{1,2} which are based on the Glauber formula for photon counting. Within this approach the relevant physical quantity to observe deviations from the poissonian statistics is the variance of the detected particles: $\text{Var}(N) = \langle N^2 \rangle - \langle N \rangle^2$. Within lowest order perturbation theory, $\text{Var}(N)$ can be expressed through the probabilities P_1 and P_2 ,

$$\text{Var}(N) = 4P_2 + P_1 - P_1^2, \quad (34)$$

$$= \langle N \rangle + 2P_2 - P_1^2. \quad (35)$$

The sign of the combination $2P_2 - P_1^2$ quantifies the deviation of the photon statistics from the poissonian result $\text{Var}(N) = \langle N \rangle$. Let us first concentrate on the regime where photons are measured in a narrow frequency band $[\Omega - \delta\Omega/2, \Omega + \delta\Omega/2]$, $\delta\Omega \ll \Omega$, far from the voltage frequency $|\omega_0 - \Omega| \sim \Omega$. According to the results of Ref. 1 the quantity $2P_2 - P_1^2$ is proportional to the measurement time $\tau = 1/\eta$, i.e., terms $\propto 1/\eta^2$ mutually cancel between the terms $2P_2$ and P_1^2 (the literal correspondence between our probabilities and those of Beenakker and Schomerus is obtained by the substitution $\gamma_0 \rightarrow 4\alpha/\Omega$). Substituting our probabilities for this regime, we find that, in contrast to the result of the Ref. 1, the leading contribution to $2P_2 - P_1^2$ is proportional to η^{-2} ,

$$2P_2 - P_1^2 \sim \frac{4\alpha^2}{\pi^2} [\delta\Omega\tau]^2 (RT)^2 \left(\frac{\omega_0 - \Omega}{\Omega} \right)^2. \quad (36)$$

Technically this difference arises from the fact, that P_1^2 cannot compensate for the contribution from the reducible current correlator in $2P_2$, see Eq. (23). The physical reason for the observed distinction between our results and the results of Ref. 1 lies in the different measurement procedure. The Glauber photodetection procedure implies a real *counting* of the photons, i.e., in addition to the number of photons one gains extra information on the time of the detection. In contrast, in our detection scheme we do not *count* the photons—our probabilities $P_1(\Omega)$ and $P_2(\Omega_1, \Omega_2)$ contain only information about the total number of the photons at the end of the measurement. Thus, we do not know which photon with frequency Ω_1 or Ω_2 was detected first. As a result, our probability to observe two photons $P_2(\Omega)$ is two times larger than that obtained via the Glauber detection scheme, and hence no compensation occurs in our case. Instead, it is the quantity $\bar{P}_2 = P_2 - P_1^2$ which exhibits the proper cancellation and provides a measure for the nontrivial correlations in the photon statistics in our analysis. The same discussion applies to the regime where photons are detected in the frequency band $[\omega_0 - \Delta\Omega, \omega_0]$, $\Delta\Omega < \omega_0/2$; our probability $P_2(\Omega)$ is again twice larger than the result in Ref. 2.

IV. MEASUREMENT OF THE FOURTH-ORDER CURRENT CORRELATORS

As mentioned in the introduction, the measurement of the photon statistics also reveals information on the current fluctuations in the QPC. In particular, the probabilities $P_1(\Omega)$ and $P_2(\Omega_1, \Omega_2)$ provide information on the second and fourth-order noise correlators at finite frequencies. Furthermore, the measurement of high-order current correlators, or alternatively, high-order transmitted charge cumulants, is a nontrivial issue. It turns out, that the measurement of the irreducible correlator \bar{P}_2 provides direct access to the fourth-order charge correlator.

The charge statistics is conveniently analyzed in a Gedanken experiment, where the transmitted charge is measured with the help of a spin-1/2 counter.^{6,18} Using the expression $\hat{Q}_t = \int_0^t \hat{I}(\tau) d\tau$ for the transmitted charge, the corresponding generating function involves the specific time ordering

$$\chi(\lambda) = \langle T_- [\exp(i\lambda \hat{Q}_t/2)] T_+ [\exp(i\lambda \hat{Q}_t/2)] \rangle, \quad (37)$$

where T_- and T_+ denote backward and forward time-ordering operators. The resulting statistics turns out to be binomial with the proper electron charge describing the transmitted carriers.⁶ The fourth-order (zero frequency) charge cumulant is given by a weighted combination of various time orderings,

$$\begin{aligned} \langle\langle \hat{Q}_t^4 \rangle\rangle &= \frac{1}{16} \int_0^t d\tau_1 d\tau_2 d\tau_3 d\tau_4 [\langle\langle T_- (\hat{I}_{\tau_1} \hat{I}_{\tau_2} \hat{I}_{\tau_3} \hat{I}_{\tau_4}) \rangle\rangle \\ &+ 4 \langle\langle T_- (\hat{I}_{\tau_1} \hat{I}_{\tau_2} \hat{I}_{\tau_3}) \hat{I}_{\tau_4} \rangle\rangle + 6 \langle\langle T_- (\hat{I}_{\tau_1} \hat{I}_{\tau_2}) T_+ (\hat{I}_{\tau_3} \hat{I}_{\tau_4}) \rangle\rangle \\ &+ 4 \langle\langle \hat{I}_{\tau_1} T_+ (\hat{I}_{\tau_2} \hat{I}_{\tau_3} \hat{I}_{\tau_4}) \rangle\rangle + \langle\langle T_+ (\hat{I}_{\tau_1} \hat{I}_{\tau_2} \hat{I}_{\tau_3} \hat{I}_{\tau_4}) \rangle\rangle], \quad (38) \end{aligned}$$

and the result assumes the form

$$\langle\langle \hat{Q}_t^4 \rangle\rangle = 2e^4 T(1-T)(6T^2 - 6T + 1) \frac{eVt}{h}, \quad (39)$$

with the poissonian limit restored in the tunneling regime.

While it is clear that this (binomial) result manifests itself when the charge transport is analyzed with a spin-1/2 counter, the question can be posed whether a more realistic experiment, e.g., the present photon detection experiment, can be used to measure this result.

Indeed, the measurement of the probability correlator $\bar{P}_2(\Omega_1, \Omega_2)$ tests the time-ordered fourth-order current correlator, reminding about the spin-1/2 detection scheme of Ref. 6, cf. Equations (7) and (38). Assuming an extended measurement where the emitted radiation is collected from a region near the QPC with a size L larger than the characteristic length Λ , the explicit calculation of the frequency integrals in Eq. (17) gives a result for $\bar{P}_2(\Omega_1, \Omega_2)$ at low frequencies $\Omega_1, \Omega_2 \ll eV/\hbar$ and $|\Omega_1 - \Omega_2| \gg \eta$, see Eq. (24), which coincides with the fourth-order charge correlator in Eq. (39),

$$\bar{P}_2(\Omega_1, \Omega_2) = \alpha^2 T(1-T)(6T^2 - 6T + 1) \frac{eV}{h\eta} \frac{d\Omega_1 d\Omega_2}{\Omega_1 \Omega_2}. \quad (40)$$

We thus conclude that for an extended measurement scheme with $L \gg \Lambda$, the probability correlator $\bar{P}_2(\Omega_1, \Omega_2)$ at low frequencies is proportional to the fourth-order charge cumulant $\langle\langle \hat{Q}_t^4 \rangle\rangle$ with $t = \eta^{-1}$.

V. CONCLUSION

We have presented a perturbative calculation of the statistics of photon emission from electrons scattered at a quantum point contact. In our analysis, we determine the probability densities p_n to find n photons with prescribed frequencies $\Omega_1, \dots, \Omega_n$ during a measuring time $t_0 \sim 1/\eta$; our perturbative calculation includes terms up to fourth order in the interaction Hamiltonian and allows us to calculate one- (p_1) and two- (p_2) photon processes. These probability densities are related to second- and fourth-order current correlators and hence measuring the photon statistics provides also information on higher order current correlators.

Our central quantity is the irreducible probability correlator $\bar{P}_2(\Omega_1, \Omega_2) = P_2(\Omega_1, \Omega_2) - P_1(\Omega_1)P_1(\Omega_2)$, which we find to provide the most valuable information if measured at different frequencies $|\Omega_1 - \Omega_2| \gg \eta$. Its sign provides information on the statistics of photon emission, which arises from a competition between bunching due to for multiphoton emission from one electron and anti-bunching imprinted onto the photons by the regular stream of incident electrons. The character of the emitted radiation can be tuned between bunched and anti-bunched by changing the voltage V and the transmission T across the QPC. Measuring the power cross-correlator in different transmission lines as done in a recent experiment¹² provides experimental access to this quantity. Within the usual photo detection scheme instead,^{1,2} the role in $\bar{P}_2 = P_2 - P_1^2$ is played by the deviation of the variance from the Poisson value, $\text{Var}(N) - \langle N \rangle = 2P_2 - P_1^2$. The discrepancy in the factor 2 in front of P_2 is a consequence of the different

measurement techniques, providing more detailed information in the photo detection scheme. This rather innocent looking difference in fact requires the definition of a different measurement quantity for the two cases of “counting” (set of single photon measurements) and “collecting” (single projection of a photon number state at the end); the former requires the analysis of $\text{Var}(N) - \langle N \rangle$, while the latter forces one to discuss \bar{P}_2 . Finally, we have shown that the irreducible correlator $\bar{P}_2(\Omega_1, \Omega_2)$ coincides (up to a scale factor) with the fourth-order charge cumulant $\langle\langle \hat{Q}_t^4 \rangle\rangle$ and, hence, provides practical access to this quantity.

ACKNOWLEDGMENTS

We thank F. Portier and K. Ensslin for discussions and acknowledge financial support by the Swiss National Science Foundation, the CTS-ETHZ, and the Russian Foundation for Basic Research under Grant No. 08-02-00767-a.

APPENDIX: FOURTH-ORDER CURRENT CORRELATOR

Using the scattering matrix approach, we first calculate the irreducible fourth-order current correlator in the time representation,

$$C(\vec{x}, \vec{t}) = \langle\langle \hat{I}(x_1, t_1) \hat{I}(x_2, t_2) \hat{I}(x_3, t_3) \hat{I}(x_4, t_4) \rangle\rangle, \quad (\text{A1})$$

and then determine the expression $Q^{(4)}(\Omega_1, \Omega_2)$ in Eq. (18).

We assume a scattering process described by energy independent scattering amplitudes r , \bar{r} , and t within a region near the QPC located at the origin $x=0$; here, r and \bar{r} denote reflection amplitudes for electrons coming from the left and right reservoirs, and t is the transmission amplitude. Linearizing the energy-momentum dispersion relation near the Fermi level, the electron current operator takes the form,

$$\begin{aligned} \hat{I}(x > 0, t) = & \frac{e}{h} \int d\epsilon d\epsilon' (T \hat{a}_\epsilon^\dagger \hat{a}_{\epsilon'} + r t^* \hat{a}_\epsilon^\dagger \hat{b}_{\epsilon'} + r^* t \hat{b}_\epsilon^\dagger \hat{a}_{\epsilon'}) \\ & + R \hat{b}_\epsilon^\dagger \hat{b}_{\epsilon'} e^{i(\epsilon - \epsilon')(t - x/v_F)/\hbar} - \hat{b}_\epsilon^\dagger \hat{b}_{\epsilon'} e^{i(\epsilon - \epsilon')(t + x/v_F)/\hbar}, \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \hat{I}(x < 0, t) = & -\frac{e}{h} \int d\epsilon d\epsilon' (R \hat{a}_\epsilon^\dagger \hat{a}_{\epsilon'} + \bar{r}^* t \hat{a}_\epsilon^\dagger \hat{b}_{\epsilon'} + \bar{r} t^* \hat{b}_\epsilon^\dagger \hat{a}_{\epsilon'}) \\ & + T \hat{b}_\epsilon^\dagger \hat{b}_{\epsilon'} e^{i(\epsilon - \epsilon')(t + x/v_F)/\hbar} + \hat{a}_\epsilon^\dagger \hat{a}_{\epsilon'} e^{i(\epsilon - \epsilon')(t - x/v_F)/\hbar}, \end{aligned} \quad (\text{A3})$$

where \hat{a}_ϵ and \hat{b}_ϵ are annihilation operators for electron scattering states incoming from the left and right reservoirs, respectively (we assume spinless electrons; $T = |t|^2$ and $R = 1 - T$ are transmission and reflection probabilities, v_F is the Fermi velocity).

The current operator $\hat{I}(x, t)$ can be written as a sum $\hat{I}(x, t) = \hat{I}^-(\xi^-) + \hat{I}^+(\xi^+)$ of outgoing and incoming currents $I^-(\xi^-)$ and $I^+(\xi^+)$ which depend only on the retarded variables $\xi^\pm = t \pm |x|/v_F$. Below, we concentrate on the current fluctuations to the right of the barrier. Introducing an additional index $\alpha = \pm$ denoting the incoming and outgoing currents, we rewrite the current operators in a compact form,

$$\hat{I}^\alpha(\xi^\alpha) = \frac{e}{h} \int d\epsilon d\epsilon' e^{i(\epsilon - \epsilon')\xi^\alpha/\hbar} \sum_{ij} \hat{c}_i^\dagger(\epsilon) A_{ij}^\alpha \hat{c}_j(\epsilon'), \quad (\text{A4})$$

where we have defined $\hat{c}_1(\epsilon) = \hat{a}_\epsilon$, $\hat{c}_2(\epsilon) = \hat{b}_\epsilon$, and the 2×2 matrices A^\pm

$$A^+ = \begin{pmatrix} T & t^* r \\ r^* t & R \end{pmatrix}, \quad A^- = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}, \quad (\text{A5})$$

defined to the right of the barrier. In order to calculate the fourth-order current correlator $C(\vec{x}, \vec{t})$, we have to average over all possible products of four current operators,

$$C^{\vec{\alpha}}(\vec{\xi}) = \langle\langle \hat{I}^{\alpha_1}(\xi_1^{\alpha_1}) \hat{I}^{\alpha_2}(\xi_2^{\alpha_2}) \hat{I}^{\alpha_3}(\xi_3^{\alpha_3}) \hat{I}^{\alpha_4}(\xi_4^{\alpha_4}) \rangle\rangle, \quad (\text{A6})$$

with $C(\vec{x}, \vec{t}) = \sum_{\vec{\alpha}} C^{\vec{\alpha}}(\vec{\xi})$. Below, we use the shorthand $\xi_i^{\alpha_i} = \xi_i$ and $A^{\alpha_i} = A_i$ and put $\hbar = 1$. Using Wick's theorem and taking averages over the reservoirs, we arrive at the expression

$$\begin{aligned} C^{\vec{\alpha}}(\vec{\xi}) = & \frac{e^4}{(2\pi)^4} \int \prod_{i=1}^4 d\epsilon_i (e^{i\epsilon_1(\xi_1 - \xi_2)} e^{i\epsilon_2(\xi_2 - \xi_3)} e^{i\epsilon_3(\xi_3 - \xi_4)} e^{i\epsilon_4(\xi_4 - \xi_1)} \text{Tr}\{N(\epsilon_1) A_1 \bar{N}(\epsilon_4) A_4 N(\epsilon_3) A_3 N(\epsilon_2) A_2\} \\ & + e^{i\epsilon_1(\xi_1 - \xi_4)} e^{i\epsilon_2(\xi_2 - \xi_1)} e^{i\epsilon_3(\xi_3 - \xi_2)} e^{i\epsilon_4(\xi_4 - \xi_3)} \text{Tr}\{N(\epsilon_1) A_1 \bar{N}(\epsilon_2) A_2 \bar{N}(\epsilon_3) A_3 \bar{N}(\epsilon_4) A_4\} \\ & - e^{i\epsilon_1(\xi_1 - \xi_4)} e^{i\epsilon_2(\xi_2 - \xi_3)} e^{i\epsilon_3(\xi_3 - \xi_1)} e^{i\epsilon_4(\xi_4 - \xi_2)} \text{Tr}\{N(\epsilon_1) A_1 \bar{N}(\epsilon_3) A_3 N(\epsilon_2) A_2 \bar{N}(\epsilon_4) A_4\} \\ & - e^{i\epsilon_1(\xi_1 - \xi_3)} e^{i\epsilon_2(\xi_2 - \xi_1)} e^{i\epsilon_3(\xi_3 - \xi_4)} e^{i\epsilon_4(\xi_4 - \xi_2)} \text{Tr}\{N(\epsilon_1) A_1 \bar{N}(\epsilon_2) A_2 \bar{N}(\epsilon_4) A_4 N(\epsilon_3) A_3\} \\ & - e^{i\epsilon_1(\xi_1 - \xi_3)} e^{i\epsilon_2(\xi_2 - \xi_4)} e^{i\epsilon_3(\xi_3 - \xi_2)} e^{i\epsilon_4(\xi_4 - \xi_1)} \text{Tr}\{N(\epsilon_1) A_1 \bar{N}(\epsilon_4) A_4 N(\epsilon_2) A_2 \bar{N}(\epsilon_3) A_3\} \\ & - e^{i\epsilon_1(\xi_1 - \xi_2)} e^{i\epsilon_2(\xi_2 - \xi_4)} e^{i\epsilon_3(\xi_3 - \xi_1)} e^{i\epsilon_4(\xi_4 - \xi_3)} \text{Tr}\{N(\epsilon_1) A_1 \bar{N}(\epsilon_3) A_3 \bar{N}(\epsilon_4) A_4 N(\epsilon_2) A_2\}, \end{aligned} \quad (\text{A7})$$

with the 2×2 matrices $N(\epsilon)$ and $\bar{N}(\epsilon)$ defined as

$$N(\epsilon) = \begin{pmatrix} n_L(\epsilon) & 0 \\ 0 & n_R(\epsilon) \end{pmatrix}, \quad \bar{N}(\epsilon) = 1 - N(\epsilon), \quad (\text{A8})$$

and $n_L(\epsilon)$ and $n_R(\epsilon)$ the Fermi distribution functions of the left and right electronic reservoirs. Next, we integrate over the energies in Eq. (A7) using

$$\int d\epsilon (1 - n_{L/R}(\epsilon)) e^{i\epsilon\xi} = \frac{i\pi\theta e^{\pm i\omega_0\xi/2}}{\sinh[\pi\theta(\xi + i\delta')]},$$

$$\int d\epsilon n_{L/R}(\epsilon) e^{i\epsilon\xi} = \frac{-i\pi\theta e^{\pm i\omega_0\xi/2}}{\sinh[\pi\theta(\xi - i\delta')]}, \quad (\text{A9})$$

where θ is the temperature of the fermionic reservoirs and $\omega_0 = eV/\hbar$ is the voltage frequency defined by the bias voltage V applied to the QPC. $\delta', \delta'' > 0$ are regularization parameters; for an energy independent transparency, they are of order $\delta' \sim \hbar/(E_c - E_F)$ and $\delta'' \sim \hbar/E_F$, where E_c is the energy width of the conduction band and E_F the Fermi energy. Below, we define a single regularization parameter $\delta = \max\{\delta', \delta''\}$ and assume the zero temperature limit $\theta = 0$. After integration and using these simplifications we obtain the expression

$$C^{\vec{\alpha}}(\vec{\xi}) = \frac{e^4}{(2\pi)^4} \left\{ - \frac{\text{Tr}[p(\xi_1 - \xi_4)A_1p(\xi_2 - \xi_1)A_2p(\xi_3 - \xi_2)A_3p(\xi_4 - \xi_3)A_4] + c.c.}{(\xi_1 - \xi_4 - i\delta)(\xi_2 - \xi_1 + i\delta)(\xi_3 - \xi_2 + i\delta)(\xi_4 - \xi_3 + i\delta)} \right.$$

$$+ \frac{\text{Tr}[p(\xi_1 - \xi_4)A_1p(\xi_3 - \xi_1)A_3p(\xi_2 - \xi_3)A_2p(\xi_4 - \xi_2)A_4] + c.c.}{(\xi_1 - \xi_4 - i\delta)(\xi_3 - \xi_1 + i\delta)(\xi_3 - \xi_2 + i\delta)(\xi_4 - \xi_2 + i\delta)}$$

$$\left. + \frac{\text{Tr}[p(\xi_1 - \xi_3)A_1p(\xi_2 - \xi_1)A_2p(\xi_4 - \xi_2)A_4p(\xi_3 - \xi_4)A_3] + c.c.}{(\xi_1 - \xi_3 - i\delta)(\xi_2 - \xi_1 + i\delta)(\xi_4 - \xi_2 + i\delta)(\xi_4 - \xi_3 + i\delta)} \right\}, \quad (\text{A10})$$

where $p(\xi)$ is the 2×2 diagonal matrix,

$$p(\xi) = \begin{pmatrix} e^{i\omega_0\xi} & 0 \\ 0 & 1 \end{pmatrix}. \quad (\text{A11})$$

Next, we consider the irreducible contribution to the probability $P_2(\Omega_1, \Omega_2)$, see Eq. (17). This contribution is proportional to the frequency integral over the fourth-order noise correlator $S^{(4)}(\omega_1, \Omega_1 + \Omega_2, \omega_2)$ with a specific kernel. Assuming a stationary situation (i.e., only relative times are relevant) and changing from frequency to time variables in Eq. (18), we have to calculate the expression

$$Q^{(4)}(\Omega_1, \Omega_2) = \int_{-\infty}^0 d\tau_1 \int_0^{\infty} d\tau_2 \tilde{C}(\tau_1, \Omega_1 + \Omega_2, \tau_2),$$

$$(e^{-i\Omega_1(\tau_1 + \tau_2)} + e^{-i(\Omega_1\tau_1 + \Omega_2\tau_2)} + \Omega_1 \leftrightarrow \Omega_2), \quad (\text{A12})$$

where we have defined

$$\tilde{C}(\tau_1, \Omega, \tau_2) = \int d\tau \tilde{C}(\tau_1, \tau, \tau_2) e^{-i\Omega\tau}. \quad (\text{A13})$$

Here, $\tilde{C}(\tau_1, \tau_2, \tau_3)$ is the coordinate averaged correlator $C(\vec{x}, \vec{t})$ expressed in terms of the relative time variables,

$$\tilde{C}(\tau_1, \tau_2, \tau_3) = \int d^4x F(\vec{x}) C(\vec{x}; \tau_1 + \tau_2 + \tau_3, \tau_2 + \tau_3, \tau_3, 0), \quad (\text{A14})$$

with the kernel $F(\vec{x}) = \prod_i f(x_i)$ describing the coupling region of the plasmon.

Next, we find the nonvanishing contributions $\tilde{C}^{\vec{\alpha}}(\tau_1, \Omega > 0, \tau_2)$ to the correlator $\tilde{C}(\tau_1, \Omega, \tau_2) = \sum_{\vec{\alpha}} \tilde{C}^{\vec{\alpha}}(\tau_1, \Omega, \tau_2)$ defined in Eq. (A13); these are identified as those with $\vec{\alpha} \in \{(\text{----}), (+\text{----}), (-\text{+---}), (\text{---+}), (\text{---+}), (\text{---+}), (\text{---+}), (\text{---+}), (\text{---+}), (\text{---+})\}$. The corresponding correlators can be written in the form,

$$\tilde{C}^{\vec{\alpha}}(\tau_1, \Omega, \tau_2) = \frac{e^4}{4\pi^3} e^{i\Omega(\tau_1 + \tau_2)/2} \int d^4x F(\vec{x}) I_{\Omega}^{\vec{\alpha}}(z_1, z_2)$$

$$\times \exp\left[\frac{i\Omega(\alpha_1|x_1| + \alpha_2|x_2| - \alpha_3|x_3| - \alpha_4|x_4|)}{2v_F} \right], \quad (\text{A15})$$

where we have introduced the new retarded variables,

$$z_1(\alpha_1, \alpha_2) = \tau_1 + \frac{\alpha_1|x_1| - \alpha_2|x_2|}{v_F},$$

$$z_2(\alpha_3, \alpha_4) = \tau_2 + \frac{\alpha_3|x_3| - \alpha_4|x_4|}{v_F}, \quad (\text{A16})$$

and the functions $I_{\Omega}^{\vec{\alpha}}(z_1, z_2)$ have the form,

$$\begin{aligned}
I_{\Omega}^{--} &= 2RT(T-R)^2\Theta(\omega_0 - \Omega) \left\{ \cos[\omega_0(z_1 - z_2)/2] \frac{\sin[(\omega_0 - \Omega)(z_1 + z_2)/2]}{(z_1 + z_2)z_1z_2} + z_2 \rightarrow -z_2 \right\} \\
&\quad + 2(RT)^2\Theta(\omega_0 - \Omega/2) \left\{ \frac{\sin[(\omega_0 - \Omega/2)(z_1 + z_2)]}{(z_1 + z_2)z_1z_2} + z_2 \rightarrow -z_2 \right\}, \\
I_{\Omega}^{+-} &= RT\Theta(\omega_0 - \Omega)e^{-i\omega_0z_1/2} \left(\left\{ (Re^{-i\omega_0z_2/2} + Te^{i\omega_0z_2/2}) \frac{\sin[(\omega_0 - \Omega)(z_1 + z_2)/2]}{(z_1 + z_2)(z_1 - i\delta)z_2} + z_2 \rightarrow -z_2 \right\} \right. \\
&\quad \left. + 2R \cos[\omega_0z_2/2] \left\{ \frac{\sin[(\omega_0 - \Omega)(z_1 - z_2)/2]}{(z_1 - z_2)z_1z_2} + z_2 \rightarrow -z_2 \right\} \right), \\
I_{\Omega}^{++} &= RT\Theta(\omega_0 - \Omega)e^{i\omega_0(z_1+z_2)/2} \left(\left\{ \frac{\sin[(\omega_0 - \Omega)(z_1 + z_2)/2]}{(z_1 + z_2)z_1z_2} + z_2 \rightarrow -z_2 \right\} - \frac{\sin[(\omega_0 - \Omega)(z_1 + z_2)/2]}{(z_1 + z_2)(z_1 - i\delta)(z_2 - i\delta)} \right), \\
I_{\Omega}^{+-} &= RT\Theta(\omega_0 - \Omega)e^{i\omega_0(z_2-z_1)/2} \left(\left\{ \frac{\sin[(\omega_0 - \Omega)(z_1 + z_2)/2]}{(z_1 + z_2)z_1z_2} + z_2 \rightarrow -z_2 \right\} + \frac{\sin[(\omega_0 - \Omega)(z_1 - z_2)/2]}{(z_1 - z_2)(z_1 - i\delta)(z_2 - i\delta)} \right).
\end{aligned}$$

The remaining functions $I_{\Omega}^{\bar{\alpha}}$ can be expressed through the four above,

$$\begin{aligned}
I_{\Omega}^{+-} &= I_{\Omega}^{+-}(-z_1, z_2, -\delta), \\
I_{\Omega}^{--} &= I_{\Omega}^{--}(-z_2, z_1, -\delta), \\
I_{\Omega}^{++} &= I_{\Omega}^{++}(z_2, z_1, \delta), \\
I_{\Omega}^{+-} &= I_{\Omega}^{+-}(-z_1, -z_2, -\delta), \\
I_{\Omega}^{+-} &= I_{\Omega}^{+-}(-z_1, -z_2, -\delta).
\end{aligned} \tag{A17}$$

Finally, we have to perform the time integrals over τ_1 and τ_2 in Eq. (A12), we regularize the divergent denominators in $I_{\Omega}^{\bar{\alpha}}(z_1, z_2)$ using the Sokhotsky formula,

$$\lim_{\delta \rightarrow 0^+} \frac{1}{z \pm i\delta} = \mathcal{P} \frac{1}{z} \mp i\pi\delta'(z), \tag{A18}$$

where the δ -function has to be understood as $\pi\delta'(z) = \delta/(z^2 + \delta^2)$ with a finite width δ . The correlator $Q^{(4)}(\Omega_1, \Omega_2)$ then takes the form

$$\begin{aligned}
Q^{(4)}(\Omega_1, \Omega_2) &= \frac{e^4}{2\pi} RT\Theta(\omega_0 - \Omega_1 - \Omega_2)(\omega_0 - \Omega_1 - \Omega_2) \int d^4x F(\vec{x}) \left\{ (T-R)^2 e^{-i(\Omega_1/v_F)(|x_1|-|x_4|)} e^{-i(\Omega_2/v_F)(|x_2|-|x_3|)} \right. \\
&\quad + \frac{T-R}{2} (e^{i(\Omega_1/v_F)(|x_1|+|x_4|)} e^{-i(\Omega_2/v_F)(|x_2|-|x_3|)} g(|x_1| + |x_2|) + e^{-i(\Omega_1/v_F)(|x_1|+|x_4|)} e^{-i(\Omega_2/v_F)(|x_2|-|x_3|)} i g(|x_3| + |x_4|) + \Omega_1 \leftrightarrow \Omega_2) \\
&\quad \left. + \frac{1}{4} g(|x_1| + |x_2|) g(|x_3| + |x_4|) (e^{-i(\Omega_1/v_F)(|x_1|-|x_4|)} e^{i(\Omega_2/v_F)(|x_2|-|x_3|)} + e^{-i(\Omega_1/v_F)(|x_1|+|x_4|)} e^{i(\Omega_2/v_F)(|x_2|+|x_4|)} + \Omega_1 \leftrightarrow \Omega_2) \right\} \\
&\quad - 2 \frac{e^4}{2\pi} (RT)^2 \Theta(\omega_0 - \max\{\Omega_1, \Omega_2\})(\omega_0 - \max\{\Omega_1, \Omega_2\}) \int d^4x F(\vec{x}) e^{-i(\Omega_1/v_F)(|x_1|-|x_4|)} e^{-i(\Omega_2/v_F)(|x_2|-|x_3|)}, \tag{A19}
\end{aligned}$$

where $g(x) = 1 - \Theta_{\delta}(x) + \Theta_{\delta}(-x)$, with $\Theta_{\delta}(x) = \int^x dy \delta'(y)$ a Heavisidelike function with a finite width $\lambda = v_F \delta$ defined by the regularization parameter δ ; with the parameters $\delta' \sim \hbar/(E_c - E_F)$ and $\delta'' \sim \hbar/E_F$, we obtain $\lambda \sim \lambda_F$ of order of the Fermi wavelength. Then $g(|x|) = 1$ for $|x| \ll \lambda$ and g vanishes in the opposite case.

For a large collection area with $\lambda < \Lambda < L$, we can drop all terms containing a factor $g(x)$ in Eq. (A19); the integration over \vec{x} generates a factor $\sim (v_F/\Omega)^4$ and we arrive at the result Eq. (23) [we approximate the factors $1/(1 + \Lambda^2/L^2) \approx 1$].

Taking into account the spin 1/2 of the electron, the above expression has to be multiplied by a factor 2. Similarly, in a multichannel situation, we have to sum the correlators $Q_n^{(4)}(\Omega_1, \Omega_2)$ for all channels n with transparency T_n .

- ¹C. W. J. Beenakker and H. Schomerus, *Phys. Rev. Lett.* **86**, 700 (2001).
- ²C. W. J. Beenakker and H. Schomerus, *Phys. Rev. Lett.* **93**, 096801 (2004).
- ³R. J. Glauber, *Phys. Rev.* **131**, 2766 (1963).
- ⁴G. B. Lesovik and R. Loosen, *JETP Lett.* **65**, 295 (1997).
- ⁵U. Gavish, Y. Levinson, and Y. Imry, *Phys. Rev. B* **62**, R10637 (2000).
- ⁶L. S. Levitov, H. Lee, and G. B. Lesovik, *J. Math. Phys.* **37**, 4845 (1996).
- ⁷*Quantum Noise in Mesoscopic Physics*, edited by Y. Nazarov, in NATO Science Series II: Mathematics, Physics and Chemistry (Kluwer, Delft, 2003), Vol. 97.
- ⁸S. Gustavsson, R. Leturcq, B. Simovic, R. Schleser, T. Ihn, P. Studerus, K. Ensslin, D. C. Driscoll, and A. C. Gossard, *Phys. Rev. Lett.* **96**, 076605 (2006).
- ⁹R. Aguado and L. P. Kouwenhoven, *Phys. Rev. Lett.* **84**, 1986 (2000).
- ¹⁰B. Küng, S. Gustavsson, T. Choi, I. Shorubalko, T. Ihn, S. Schön, F. Hassler, G. Blatter, and K. Ensslin, *Phys. Rev. B* **80**, 115315 (2009).
- ¹¹J. Gabelli, L.-H. Reydellet, G. Fève, J.-M. Berroir, B. Plaçais, P. Roche, and D. C. Glattli, *Phys. Rev. Lett.* **93**, 056801 (2004).
- ¹²E. Zakkajani, J. Segala, F. Portier, P. Roche, D. C. Glattli, A. Cavanna, and Y. Jin, *Phys. Rev. Lett.* **99**, 236803 (2007).
- ¹³C. C. Grimes and G. Adams, *Phys. Rev. Lett.* **36**, 145 (1976).
- ¹⁴S. J. Allen, Jr., D. C. Tsui, and R. A. Logan, *Phys. Rev. Lett.* **38**, 980 (1977).
- ¹⁵P. J. Burke, I. B. Spielman, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, *Appl. Phys. Lett.* **76**, 745 (2000).
- ¹⁶M. J. M. de Jong and C. W. J. Beenakker, in *Mesoscopic Electron Transport*, edited by L. L. Sohn, L. P. Kouwenhoven, and G. Schön (Kluwer, The Netherlands, 1997).
- ¹⁷O. N. Dorokhov, *JETP Lett.* **36**, 318 (1982).
- ¹⁸S. Bachmann, G. Graf, and G. Lesovik, *J. Stat. Phys.* **138**, 333 (2010).